Parallelization Approaches for the Time-efficient Simulation of Hybrid Dynamical Systems: Application to Combustion Modeling

Abir Ben Khaled (IFPEN)
Mongi Ben Gaid (IFPEN)
Daniel Simon (INRIA and LIRMM-CNRS-Université Montpellier Sud de France)

EOOLT 2013 – April, 19th 2013

Motivations

- Potential improvements in computing speeds are provided by multi-core chips and parallelism
- The efficient numerical integration of systems described by equation oriented languages requires the ability to exploit parallelism
- Hardware-in-the-loop for reaching real-time
- Many applications (Engine simulating in IFPEN)

Outline

- Motivations
- Related work
- Problem Formalization
- Parallelization across the model : speedup vs. numerical errors
- Methods of model analysis and automatic parallelization
- Case-study

Related work

- Waveform relaxation [Lelarasmee *et al.*]
- Transmission line modeling [Sjölund *et al.*]
- Parallelization across the solver : PVODE [Byrne *et al.*]
- Parallelization across the model [Faure *et al.*, Ben Khaled *et al.*]

Problem Formalization

- State evolution and output equations

$$
\begin{aligned}
\dot{X} &= f(t, X, D, U) \quad \text{for } t_n \leq t < t_{t+1}, \\
Y &= g(t, X, D, U).
\end{aligned}
$$

- Event detection and location

$$h(t, X, D, U) = 0.$$

- Event handling

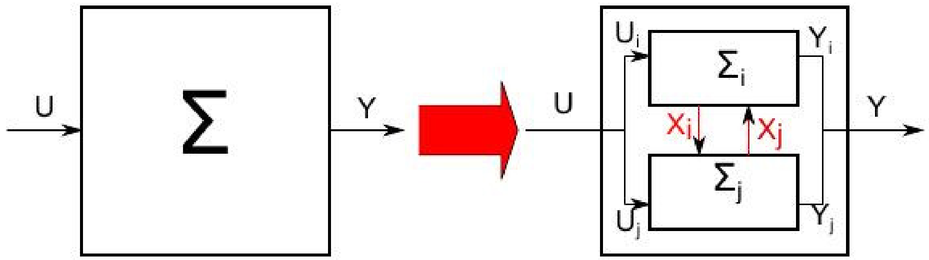
$$X(t_n) = I(t_n, X, D, U),$$

$$D(t_n) = J(t_{n-1}, X, D, U),$$

Requirements from previous experience

- Achieving good load balancing and avoiding idle time (adequate task scheduling)
- Minimization of the number of solvers interrupts due to discontinuities unrelated with the local sub-system (adequate model decomposition)
- Keeping an acceptable level of numerical stability and accuracy (adequate numerical methods)

Parallelization across the model



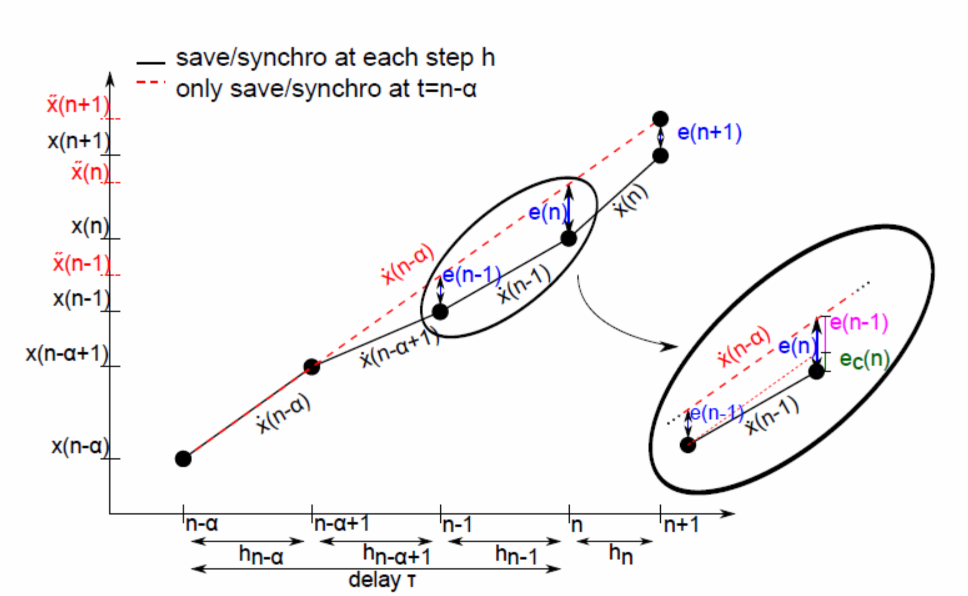
$$\begin{cases} \dot{x}_i & = & f_i(x_i, x_j, U_i) \\ Y_i & = & g_i(x_i, x_j, U_i) \end{cases} \quad \textit{and} \quad \begin{cases} \dot{x}_j & = & f_j(x_j, x_i, U_j) \\ Y_j & = & g_i(x_j, x_i, U_j) \end{cases}$$

Minimize the interaction between parallel branches

Clusters of computations with slacken synchronization

- Strongly dependent state variables
- Localized events location
- Avoid unrelated interrupts

Evaluation of delay errors



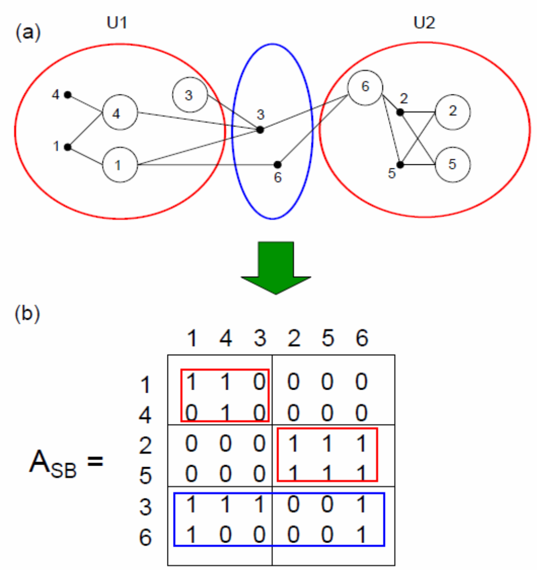
Delay errors depends on

- Integration step-size and delay
- System stiffness (including discontinuities)
- State variable dependencies

Feedback from previous work

- Intuitive partitioning is not effective for real cases
- Need for tools allowing the analysis of complex systems for the purpose of parallelization
- Investigating methodologies for automatic parallelization

Automatic partitioning using PaToH



(a) Row-net hypergraph representation of the matrix A and 2-way partitioning of it; (b) 2-way SB form of A induced by (a)

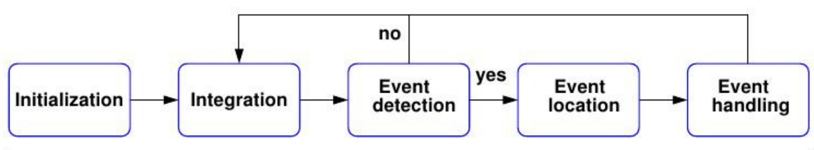
Exploitation of structural properties of some well chosen incidence matrices (states and/or events)

Automatic partitioning using PaToH

Exploitation of structural properties of some well chosen incidence matrices (states and/or events)

$$J = \begin{pmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_N} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_N} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\partial f_N}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \cdots & \frac{\partial f_N}{\partial x_N} \end{pmatrix}$$

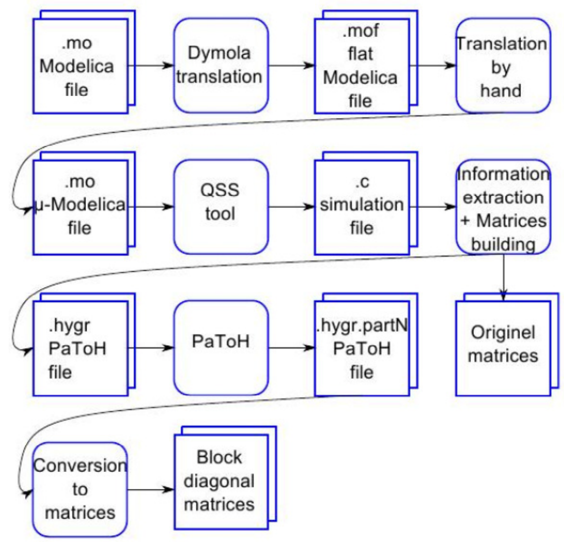
Sensitivity of X_i / X_j



Make events location as local as possible

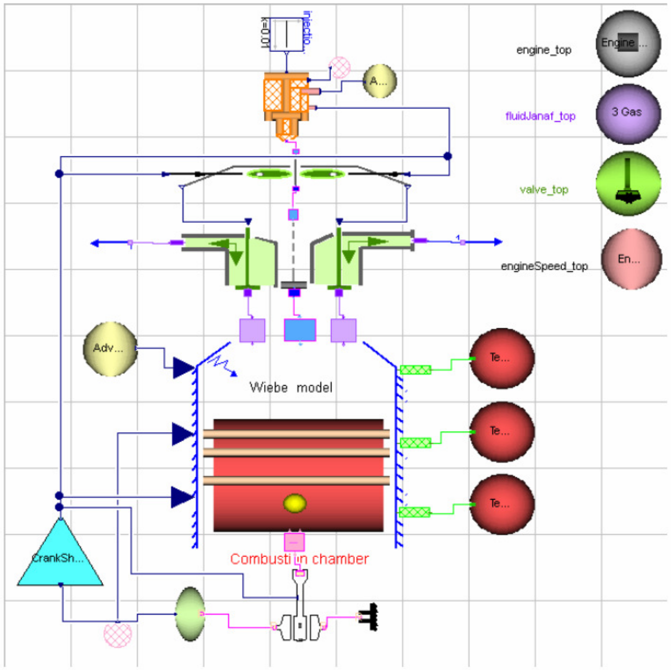
Software toolchain

Automatic extraction of relations between states and events



Case-study presentation

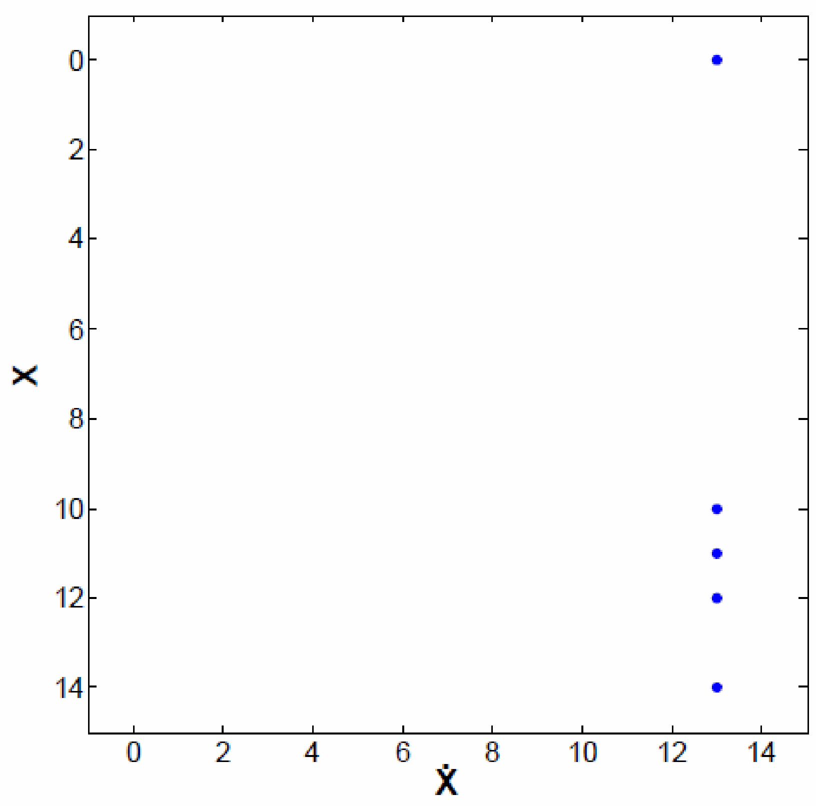
- Mono-cylinder engine
- Written in Modelica
- Translated to µ-Modelica
- Wiebe combustion model
- Characteristics
  - State variables: $n_X = 15$
  - Events: $n_Z = 111$
  - Discrete variables: $n_D = 93$



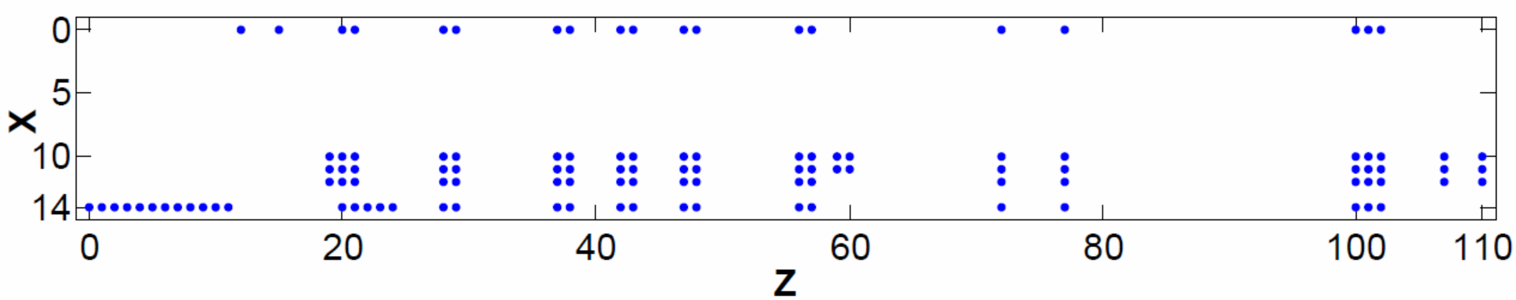
Case-study state variables

ID	Name	Details
X_0	CrankAngle	Crank Shaft angle
$X_{1\to3}$	mEvapo[3]	Gas mass evaporated due to injection in global Mass balance equation
X_4	qvAlfa	Current released heat generated by the combustion process
X_5	mrefAlfa	Burned mass fraction during combustion process
X_6	combuHeatRelease	Output current combustion heat released
$X_{7\to9}$	mCombu[3]	Gas mass derivatives due to combustion in global Mass balance equation
$X_{10\to12}$	M[3]	Mass of gas
X_{13}	Energy	Energy contained in the cylinder
X_{14}	cylinderTemp	Output temperature in the cylinder

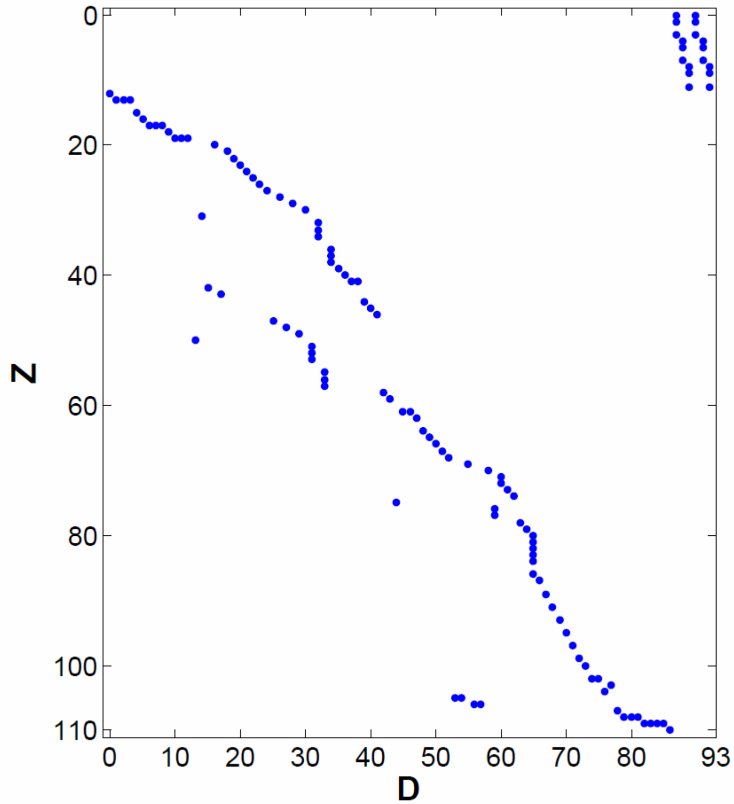
Derivatives of state variables X' depending on state variables X



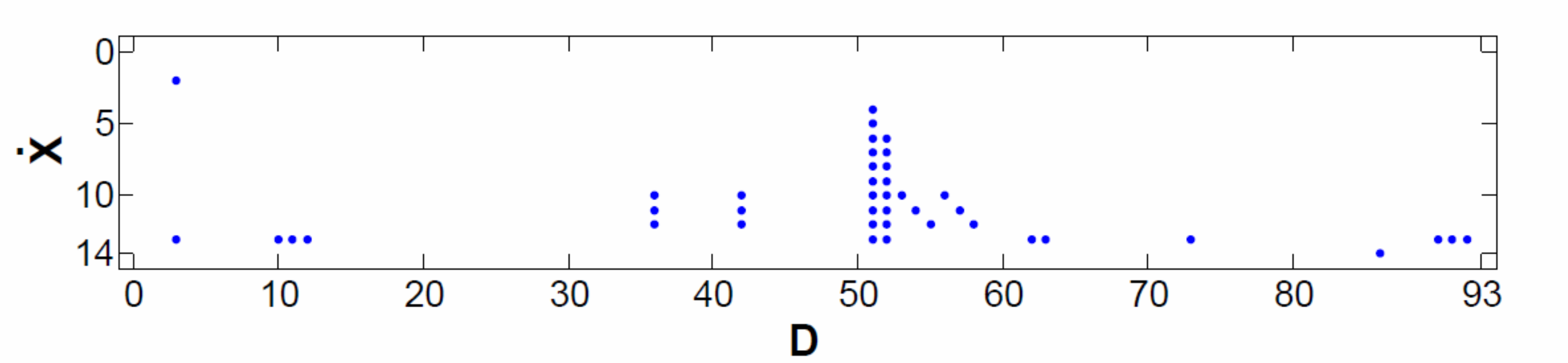
Events Z depending on state variables X



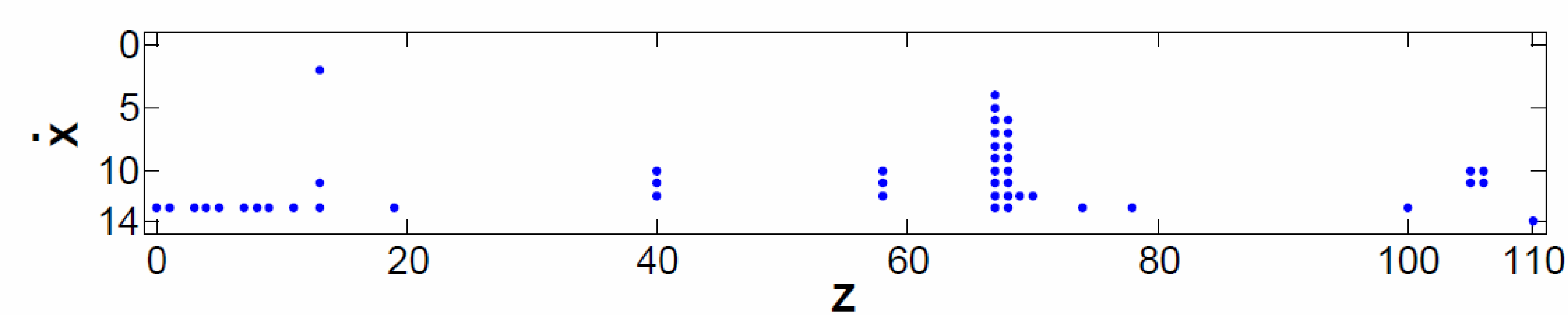
Discrete variables D influenced by events Z



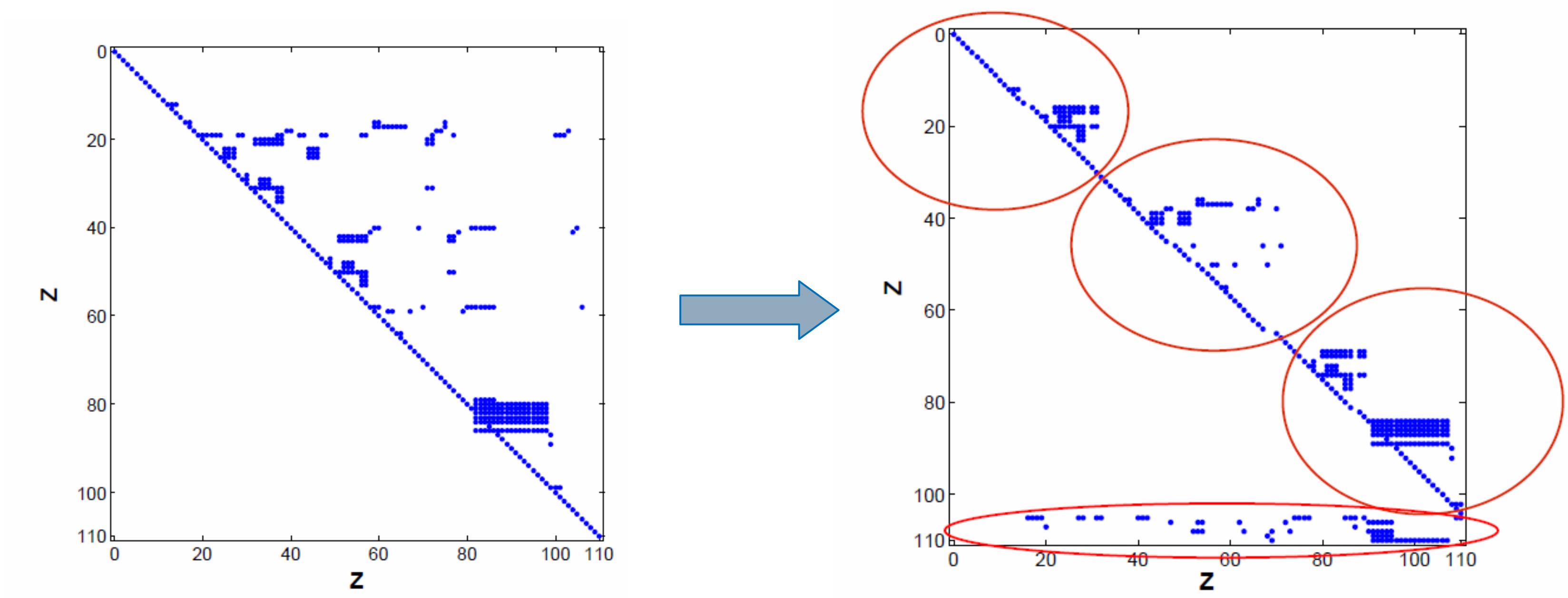
Derivatives of state variables X' influenced by discrete variables D



Derivatives of state variables X' influenced by events Z

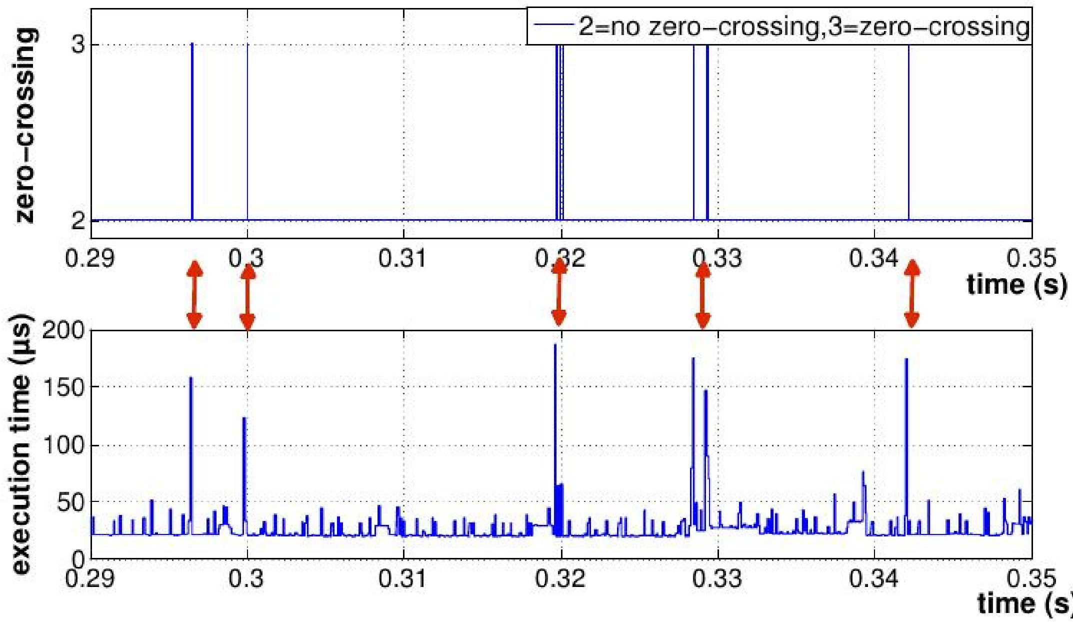


Application of PaToH to event incidence matrix



This one gives an exploitable structure!

Zero-crossings execution cost



Conclusions

- Theoretical analysis of the impact of slackened synchronization
- Toolchain for the analysis of hybrid dynamical systems for the purpose of their parallelization
- Candidate methods for automatic parallelization
- Illustration through a case-study

Future work

- Developing a parallel runtime
- Evaluation of the achievable speedup on the presented case-study
- Application to 4-cylinder model
- Application to human arm/hand simulation